Conditional Copula for Change Detection on Heterogeneous SAR Data

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***Abstract*— A new preprocessing technique is presented in this paper to perform automatic change detection in multitemporal multimodal remotely sensed images, mainly Synthetic Aperture Radar (SAR) ones. This technique is dedicated to the case where the two acquisitions, before and after an major disaster, are different for some reason (different sensor, modality of acquisition or climatic conditions).**

A measure, based on the local statistics of the images between the two dates, has proved to be a relevant change indicator. Nevertheless, the measure is valid when the two observations have been acquired with a similar point of view only. When the modalities of acquisition differ, local statistics tend to be too different, from one image to the other, to be relevant to the ground evolution without mixing to the normal changes. The technique, that overcomes this constraint, is based on the assumption that some dependence exists indeed between the two images. This dependence is modelled by the copula theory and used to perform an estimation of the local statistics that would have been observed if the modality of the first image had been similar to the other. It yields an estimation of local statistics of the first image, through the point of view of the latter. Then, usual comparison of those statistics may be applied to perform change detection.

Some results are shown on a pair of ERS images and pairs of SPOT/ERS acquired before and after a flood.

I. INTRODUCTION

Remote sensing plays an important role in disaster management. It allows the detection of changes occurring after a natural or anthropic disaster, as soon as a rapid mapping of the impacted zone may be performed [1]. For such a operational use of remote sensing data, the first available acquisition after the event has to be used whatever its modality. The *before* image is issued from available archive data. Unfortunately, those two acquisitions differ in their modality: orbits may be ascending and descending, parameters of acquisitions may differ from one image to the other even when the two acquisitions are issued from the same sensor.

Another difficulty arise since there is no model associated to the change: no model exists to predict value of pixels after a disease. When focusing on the use of SAR data, many approaches have been proposed by using local statistics comparisons since pixel-wise comparison is not robust enough. In [2], a probability density functions (pdfs) comparison has been proposed to perform a low complexity change indicator.

This point of view remains valid when the two observations are similar, but may become wrong when the two observations differ. In this paper, a preprocessing technique is proposed to make the link, in a statistical point of view, between the two observations in order to make possible the use of classical change detector even when the two observations differ. The basic idea is to process a simulated pdf that correspond to the observation that would have been *before* the event if it were acquired in the same condition as the *after* image.

II. PROBLEM FORMULATION

Two co-registered SAR intensity images I_1 and I_2 are considered at two different dates t_1 and t_2 respectively. The objective is to draw a map representing the changes that occurred in the scene between t_1 and t_2. The process is applied for each position of the current pixel. At a given position (i, j) of the images *before* and *after*, two co-located sample sets $\{x_{1;1}, \ldots, x_{1;N}\}$ and $\{x_{2;1}, \ldots, x_{2;N}\}$ are considered from the local neighborhood of the pixel at position (i, j). They are assumed to be some realizations of random variables (RVs) X_1 from image *before* and X_2 from image *after*. Let f_{X_1} (resp. f_{X_2}) be the probability density function (pdf) of the RV X_1 (resp. X_2).

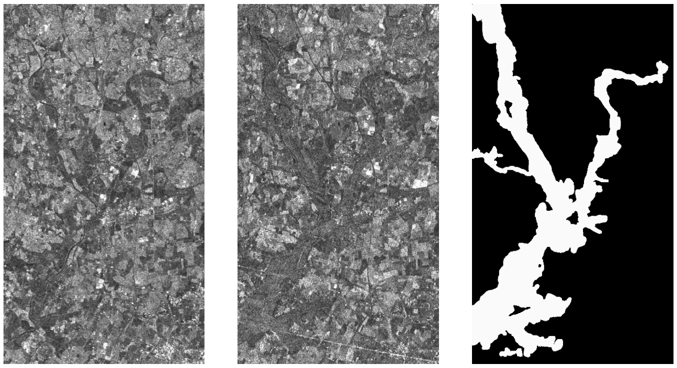
As stated in [3], the local statistics comparison is performed by using a symmetric version of the Kullback-Leibler distance:

$$\mathcal{K}(X_1, X_2) = \mathcal{K}(X_1|X_2) + \mathcal{K}(X_2|X_1) \qquad (1)$$

$$\text{with} \qquad \mathcal{K}(X_2|X_1) = \int_{\mathbb{R}} \log \frac{f_{X_1}(x)}{f_{X_2}(x)} f_{X_1}(x) dx.$$

By using this point of view, relevant change indicator may be built up by using parametric or non-parametric models in the estimation of f_{X_1} and f_{X_2}. Here, the evaluation of the pdfs and then the Kullback-Leibler distance have been optimized by using an Edgeworth series expansion, as stated in [2].

Unfortunately, this approach is not reliable when the two observations differ. Fig. 2-(a) shows the results of this measure applied on a set of ERS images acquired before and after a flood (see original images on fig. 1). Those two images are becoming very different from a statistical point of view since ground humidity induced a lack of backscattering; also, a strong wind induces surface roughness, and so texture, on the



before image I_1 10/21/1999 — *after* image I_2 10/21/2000 — ground truth

Fig. 1. Pair of ERS images acquired over Gloucester before and during a flood that occurs in October 2000.

submerged areas. The disappointing results of fig. 2-(a) may be formalized by the fact that the log-likelihood ratio may not be the good choice since f_{X_2} always differs from f_{X_1} whatever the changes. The idea here is to build a statistical formulation to yield a new pdf $f_{X_1'}$ to be used, instead of f_{X_1} to perform the comparison with f_{X_2}. In some sense, X_1' is a simulated process that express what should be the X_1 observation through the modality of I_2.

III. Dependence and copulas

The Copula theory is used to characterize the joint distribution $F_{X_1,X_2}(x_1, x_2)$ without the interaction of $F_{X_1}(x_1)$ and $F_{X_2}(x_2)$.

A. Overview on copulas

A bivariate copula is any cumulative density function on the unit square with uniform marginal functions [4]:

$$C(u_1, u_2) = \Pr(U_1 \leqslant u_1, U_2 \leqslant u_2).$$

Sklar has shown that the link between any continuous joint law F_{X_1,X_2} and its marginal laws F_{X_1}, F_{X_2} is achieved with a copula:

$$F_{X_1,X_2}(x_1, x_2) = C\Big(F_{X_1}(x_1), F_{X_2}(x_2)\Big). \tag{2}$$

As a matter of example, the copula that characterizes independence between two RVs is: $C^{\perp}(u_1, u_2) = u_1\, u_2$.

B. Empirical Copula

In a similar way to 1D pdf estimation by histogram, an *empirical copula* may be estimated by following the P. Deheuvels technique [4]. Considering N pairs of samples $\{(x_{1;\ell}, x_{2;\ell})\}_{0<\ell\leqslant N}$, the empirical copula is defined by:

$$\widehat{C}\left(\frac{n}{N}, \frac{m}{N}\right) = \frac{1}{N}\sum_{\ell=1}^{N} \mathbb{1}_{\{r_{\ell,1}\leqslant n,\, r_{\ell,2}\leqslant m\}}$$

where $r_{\ell,1}$ and $r_{\ell,2}$ are the rank statistics of $\{x_{1;\ell}\}_\ell$ and $\{x_{2;\ell}\}_\ell$. Unfortunately, it has also the same drawback as the histogram approach: it requires significant number of training sample for an accurate estimation. Hence, parametric models may be required.

C. Parametric Copula

Many copulas exist and may be used to characterize the dependency that may be found between a pair of two images. We are focusing on the family of Archimedian copulas only since one of those, the so-called Ali-Mikhail-Haq copula [5], fits the dependency of our pair of RVs.

The Archimedian copulas are a flexible family of copulas since they are defined by a 1D continuous decreasing convex function $\varphi(\cdot)$ from $[0, 1]$ to $[0, \infty[$ such that $\varphi(1) = 0$,

$$C(u_1, u_2) = \varphi^{-1}\Big(\varphi(u_1) + \varphi(u_2)\Big). \tag{3}$$

For instance, the Ali-Mikhail-Haq copula is generated by $\varphi_\theta(t) = -\ln\frac{1-\theta(1-t)}{t}$, $-1 \leqslant \theta < 1$ and takes the following expression:

$$C(u_1, u_2) = \frac{u_1 u_2}{1 - \theta(1-u_1)(1-u_2)}. \tag{4}$$

D. Parameter estimation

For most parametric copulas, their parameter depends on the *Kendall's* τ which is a concordance-discordance rank statistics. An empirical estimator of τ, given in [6], makes the link between observations and the model. When using Archimedian copulas, Kendall's τ becomes:

$$\tau = 1 + 4\int_0^1 \frac{\varphi(t)}{\varphi'(t)}dt,$$

which is approximated for Ali-Mikhail-Haq copula:

$$\tau \approx \frac{8}{9}\frac{\theta}{4-\theta} + \frac{8}{15}\left(\frac{\theta}{4-\theta}\right)^3 + \cdots \tag{5}$$

E. Choice of the best copula

In [7], authors have proposed a rank-based graphical tool for visualizing dependence between two RVs. It has been applied on polarimetric data [8]. It appears that the Ali-Mikhail-Haq copula fits the initial data much better than any other copulas known by the authors: Product, Ali-Mikhail-Haq, Farlie-Morgenstern, Frank, Gumbel, Heavy Right Tail, Clayton, Marchal-Olkin, Plackett, Raftery, Cubic and the Normal copula.

IV. The use of copula for quantile regression

A. Conditional distribution

Copula may be used for characterizing the conditional distribution, required for quantile regression. By considering the quantile $q = F_{X_2|X_1}(x_1, x_2) = \Pr(X_2 \leqslant x_2 | X_1 = x_1)$ and using uniform distribution by the transformation $u_i = F_{X_i}(x_i)$: $\Pr(X_2 \leqslant x_2 | X_1 = x_1) = \Pr(U_2 \leqslant u_2 | U_1 = u_1) = \frac{\partial C}{\partial u_1}(u_1, u_2) = C_{\partial 1}(u_1, u_2)$.

Then, a conditional distribution becomes the derivation of the copula, and so, the q-th copula quantile curve of x_2 conditional on x_1 is [9]:

$$q = F_{X_2|X_1}(x_1, x_2) = C_{\partial 1}\left(F_{X_1}(x_1), F_{X_2}(x_2)\right).$$

Hence, when $C_{\partial 1}(\cdot, \cdot)$ is invertible over the first parameter, one can tackle the copula quantile regression of X_2 on X_1.

Let $C_{\partial 1}^{\text{Inv }2}(\cdot,\cdot)$ be the inverse of $C_{\partial 1}(\cdot,\cdot)$ over the second term, the copula quantile regression becomes:

$$\begin{aligned} u_2 &= C_{\partial 1}^{\text{Inv }2}(u_1, q) \\ \text{or} \quad x_2 &= F_{X_2}^{-1}\left(C_{\partial 1}^{\text{Inv }2}(F_{X_1}(x_1), q)\right). \end{aligned} \tag{6}$$

But we are more interested in the pdf of the expected x_2 instead of their regression over some quantile value q. Hence, eq. (6) has to be applied by looking at different values of q.

By using any Archimedian copula, generated by the function $\varphi(\cdot)$:

$$q = C_{\partial 1}(u_1, u_2) = \frac{\varphi'(u_1)}{\varphi'\left(\varphi^{-1}\left(\varphi(u_1) + \varphi(u_2)\right)\right)}$$

where $\varphi'(\cdot)$ stands for the derivation of $\varphi(\cdot)$. The quantile function of eq. (6) becomes then:

$$u_2 = \varphi^{-1}\left(\varphi\left(\varphi'^{-1}\left(\frac{1}{q}\varphi'(u_1)\right)\right) - \varphi(u_1)\right). \tag{7}$$

B. Multisensor regression

Let us consider X_1 from which the distribution function $F_{X_1}(x)$ may be estimated in some way (by means of a histogram, through a parametric model or a cumulant-based expansion). The copula $C(u_1, u_2)$ has to be known by an estimation performed over a training area where no change occurs. A generic distribution function $F_2(x)$ has to be estimated also from the no-change area, on the *after* image I_2. This distribution may differ from the local $F_{X_2}(x)$ especially if a change occurs.

- The set $\{u_{1;\ell}\}_\ell$ is defined from $\{x_{1;\ell}\}_\ell$ by $u_{1;\ell} = F_{X_1}(x_{1;\ell})$, $1 \leqslant \ell \leqslant N$.
- Let the set of quantile values $\{q_\ell\}_\ell$ be generated randomly to have uniform pdf on $[0, 1]$.
- $u_{2;\ell}$ chosen such that $q_\ell = \Pr\left(U_2 \leqslant u_{2;\ell} | U_1 = u_{1;\ell}\right)$. Hence : $u_{2;\ell} = C_{\partial 1}^{\text{Inv }2}(u_{1;\ell}, q_\ell)$.
- The pdf of X_1' may be estimated by the pdf of the set of samples $\{x'_{1;\ell}\}_\ell$ defined by

$$x'_{1;\ell} = F_2^{-1}\left(C_{\partial 1}^{\text{Inv }2}(F_{X_1}(x_{1;\ell}), q_\ell)\right) \tag{8}$$

X_1' has the same *meaning* as X_1 but with its statistics issued from I_2. For change detection purpose, X_1 and X_2 are not compared anymore by using eq. (1), but rather X_1' and X_2. Hence, the multimodal change detection to be used instead of eq. (1) is:

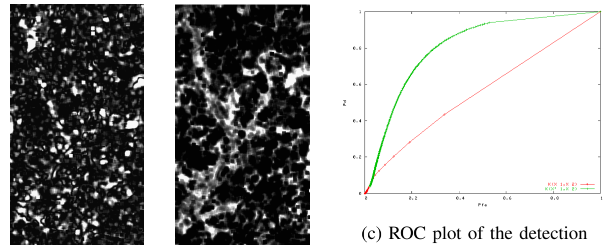
$$\begin{aligned} \mathcal{K}'(X_1, X_2) &= \mathcal{K}(X_1', X_2) = \mathcal{K}(X_1'|X_2) + \mathcal{K}(X_2|X_1') \\ \text{with} \quad X_1' &= F_2^{-1}\left(C_{\partial 1}^{\text{Inv }2}(F_{X_1}(X_1), U)\right), \end{aligned} \tag{9}$$

where U stands for a RV of uniform pdf.

In fact, a more complex similarity measure may be defined as:

$$\begin{aligned} \mathcal{K}'(X_1, X_2) = \mathcal{K}(X_1'|X_2) &- \mathcal{K}(X_1|X_1') \\ &+ \mathcal{K}(X_2'|X_1) - \mathcal{K}(X_2|X_2'). \end{aligned} \tag{10}$$

As soon as change detection between two dates is a symmetric task, it is more interesting to consider the estimation of X_2 in the point of view of image I_1 also. Then, the initial $\mathcal{K}(X_1'|X_2)$ should be completed by $\mathcal{K}(X_2'|X_1)$ which uses the same conditional copula as eq. (9) if the copula is symmetrical. The terms $\mathcal{K}(X_1|X_1')$ and $\mathcal{K}(X_2|X_2')$ of eq. (10) allow to reduce some false alarms that take their origin in the spatial variability of the images. Indeed, X_1' is yielded by a sampling through the probability function $F_2(\cdot)$. Some false alarms may appear on $\mathcal{K}(X_1'|X_2)$ if the local $F_{X_2}(\cdot)$ differs from the global $F_2(\cdot)$ even if no changes occur. The term $\mathcal{K}(X_1|X_1')$ balances false alarms on those cases since $\mathcal{K}(X_1'|X_2) - \mathcal{K}(X_1|X_1')$ may vanish if no change occurs.



(a) $\mathcal{K}(X_1, X_2)$ on a 35×35 window

(b) Detection by using eq. (10)

(c) ROC plot of the detection

Fig. 2. Change detection results by using quantile regression.

V. Application to real case

Quantile regression has been applied on a set of images: a pair of ERS images and a pair of SPOT Xs-ERS images, corresponding to a flood over Gloucester, U.K. from October and November 2000.

A. Change detection with a pair of ERS images

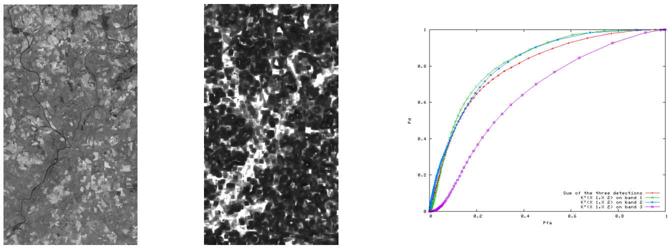
Although the pair of ERS images (of size 1318×2359) has been acquired with the same sensor, the variability of the ground in-between the two images is strong enough to be treated as a pair of heterogeneous data. In fact, fields full of water have a radiometric response very different from the first image to the latter; also, a strong wind, that had generated waves on the river, induced a high radiometric response on the submerged areas.

Fig. 2 shows the results of change detection by using quantile regression of eq. (9). Comparison of fig. 2-(a) and 2-(b) shows that the use of quantile regression yields a more tractable result. Fig. 2-(c) shows the ROC plots by using ground truth of fig. 1. It shows the significant improvement in the use of quantile regression. 80% of good detection may be reached for 30% of bad detection.

B. Change detection with a SPOT Xs-ERS pair

For this last experiment, a pair of SPOT Xs and ERS images has been considered. The ERS *after* image is the same as the one used on section V-A. Hence, the ground truth is also the one used on fig. 1. The *before* image has changed: it is a multispectral SPOT HRV image acquired on 09/05/1999, as shown on fig. 3-(a).

It is worth noting that in the case of multisensor purpose, the initial point of view of the local statistics comparison of eq. (1) may not be usable anymore. In the present case, the



(a) *before* SPOT images image 09/05/1999

(b) sum of $\mathcal{K}'(X_1, X_2)$ on each band

(c) ROC plot of the detection for band 1, 2 and 3 of the SPOT image.

Fig. 3. Change detection results by using quantile regression on a SPOT-ERS pair. The overall detection is performed by adding the detections of the three bands.

initial global statistics of the *before* and *after* observations are different (the supports of the histograms are disjointed), then changes are detected almost everywhere — in fact, $\mathcal{K}(X_1, X_2)$ is almost not-a-number. Even with any pre-processing technique, the change detection results of eq. (1) are closed to a random decision since $P_d \approx P_{fa}$. The use of quantile regression helps the detection although a lot of false alarms remain.

In order to deal with multispectral data and to make the link with ERS data, a copula has been used for each spectral band. So that, as many quantile regressions as the number of spectral bands have been performed independently. The change detection of eq. (9) has been applied to detect changes between band 1 (Green: 0.5–$0.59\,\mu$m) of the SPOT image and the ERS image. It has been also applied on band 2 (Red: 0.61–$0.68\,\mu$m) of the SPOT image and the ERS image, and between band 3 (NIR: 0.79–$0.98\,\mu$m) and the ERS image. The three detections are added to yield one result only, and shown on fig. 3.

The same training area has been used for the estimation of the copulas. For this case, the Gumbel copula has been found to fit better the dependence between the observations. Unfortunately, no tractable expression of the inverse of this copula has been found. Then, the empirical copula has been used instead.

It is of interest to stress that the use of band 1 and 2 yields better results than band 3. The NIR band is more sensitive to the chlorophyll and soil moisture. Hence, band 3 image appears to be more contrasted than band 1 (Green) and 2 (Red). For this application, band 3 should be more interesting in an optic to optic comparison, nevertheless the link between band 3 and SAR observation is more heterogeneous than for band 1 or 2. In that particular case, the use of band 1 alone give an accuracy of $P_d = 80\%$ with $P_{fa} = 30\%$ (equivalent results are found with band 2). This thematic justification may not be valid for all cases. It should be more interesting to have an *a priori* quantitative indicator of the effectiveness of the quantile regression.

VI. Conclusion

Quantile regression has been used to overcome multisensor variability problems in change detection. The change detection does not compare anymore the local statistics of the two images but a simulated version of the initial observation by considering the point of view (in a statistical sense) of the second observation.

ROC plots proved the validity of this point of view.

Quantile regression has been implemented by using parametric or non-parametric models taken from the copulas theory. The Ali-Mikhail-Haq copula have been found to fit the best the pair of ERS images used for illustration. No parametric copulas have been found to fit SPOT Xs - ERS pair while being usable for quantile regression implementation. Copulas are an interesting tool for such a quantile regression since the multisensor regression does not depend on the local marginal pdfs. Hence, it can be used everywhere on the image whatever the local ground characteristics. Nevertheless, parametric copulas used may not fit well enough the dependency between the two images. The use of empirical copula required representative training samples to prevent from coarse estimations and then poor change detection results.

When the resolution of the two sensors is becoming different, there is no guaranty that the pdf comparison point of view be appropriate for change detection purpose anymore. A careful analysis should be achieved for this particular case.

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