Estimation of Sea-Ice SAR clutter statistics from Pearson's system of distributions

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Abstract—**SAR images can be used to help ship routing in sea-ice conditions. In this study, we focus on the Antarctic region where no multi-year ice nor big ice floes are to be found. As a matter of fact, each clutter obeys to a backscattering mechanism that induces a specific pixel distribution and our attempt is to identify automatically the correct distribution for each ice type. The problem is that of generalized mixture estimation and unsupervised image classification. In this work, we modelled the mixture with distributions from the Pearson's system. Parameters estimation is realized according to the ICE algorithm in the context of hidden Markov chains. The results obtained from the Pearson's system are compared to the ones obtained with a classical mixture of Gaussian distributions.**

I. Introduction

SAR IMAGES can be used to help ship routing in sea-ice conditions. The knowledge of sea-ice characteristics is of importance in order to find the most appropriate route for a given ship. In this study, we focus on the Antarctic region where no multi-year ice nor big ice floes are expected to be found. Ice types are mostly pancakes, brash or small ice floes. A typical example is given is Fig. 1. The image histogram shape (see the vignette in Fig. 4) does not show obvious thresholds. Thus, remotely sensed images should not be analyzed using structural methodology but with the help of statistical modelling.

In the context of SAR image processing, Gaussian distributions remain the most used ones. In order to take into account theoretical results in the modelling of backscattering mechanisms, a number of candidate families has been presented previously, such as Gamma distributions [1], K distributions (based on the modified Bessel function of the second kind) [2], [3] and Beta distributions [4]. The interest of these distribution functions comes essentially from the large variety of possible shapes that can be obtained by modifying a limited number of parameters (up to four). In particular, all of them can take into account the dissymmetry of class densities, which is not the case of Gaussian densities. Another interesting point is that some of them have a finite or semi-finite support, which is of great interest in SAR image processing. In order to enlarge the set of available shapes, one solution is to consider, not only one of the families cited above, but all of them in a unified way with the help of the Pearson's system of distributions [5].

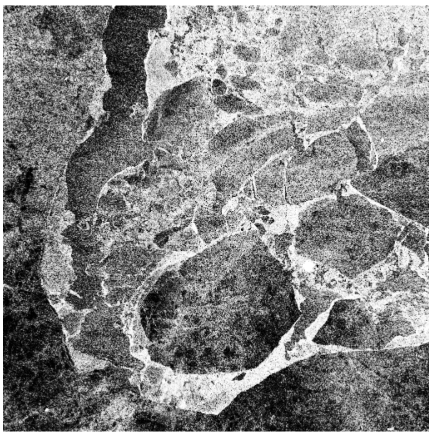


Fig. 1. SAR image of sea-ice used for experimentation. The image gray-levels have been equalized for better visualization of clutters. ©IFRTP - RadarSat-SW - 512×512 (October 24, 1996 - Dumont d'Urville sea).

As a matter of fact, each clutter obeys to a backscattering mechanism that induces a specific class distribution and our attempt is to identify automatically the correct family of distributions, from the Pearson's system, of each ice type. We thus seek both the nature of the corresponding distribution and the parameters that best describe its samples. The problem is that of generalized mixture estimation and unsupervised image classification. From the large variety of methods (including EM and SEM), we chose the *Iterated Conditional Estimation* (ICE) algorithm proposed by W. Pieczynski [6] in the context of *Hidden Markov Chains* (HMC) [7], [8]. The HMC framework exploits the spatial dependencies between neighboring pixels and impose a spatial regularity constraint on the classes. It is also a substantially quicker alternative to hidden Markov fields.

The entire algorithm is summarized in section II and illustrated in section III for the image in Fig. 1. The results obtained from the generalized mixture estimation are compared to the ones obtained with a classical Gaussian mixture. Conclusions and future work are drawn in section IV.

II. Sea-ice clutters distribution estimation

This section is not intended to give a theoretical justification, neither a complete description, of the ICE algorithm in the context of Hidden Markov Chains. Interested readers may consult [8], [9]. We only briefly sum up the entire algorithm and give an overview of the Pearson's system of distributions.

A. Description of the algorithm

The 2D original image is modelled as a 1D chain using a Hilbert-Peano scan. We consider two sets of random vectors:

$X = (X_s)_{s \in S}$ and $Y = (Y_s)_{s \in S}$ where S is the set of image pixels. Each X_s takes its value in a finite set of classes $\Omega = \{\omega_1, \ldots, \omega_K\}$, and Y_s takes its value in $\mathbb{R}$ (gray-levels). The segmentation problem is to estimate the unobserved realization $X = x$ from the observed realization $Y = y$, where $y = (y_s)_{s \in S}$ is the image to be segmented. In the context of unsupervised bayesian segmentation, we need to estimate all the parameters defining the distribution of (X, Y), i.e. P_X the distribution of X, and the family $P_Y^{X=x}$ of the distributions of Y conditional to X. Let us denote by α all the parameters concerning P_X, and by $\beta = (\beta_1, \ldots, \beta_K)$ all the parameters defining the family $P_Y^{X=x}$ $(\theta = (\alpha, \beta))$. In the case of HMC, α is given by the initial state and the transition matrix of the homogeneous Markov chain. In classical mixture estimation, densities $P_Y^{X=x}$ are Gaussian and β is composed by means and variances.

From the observed image Y to the segmented image X, the algorithm can be decomposed into three main phases:

Initialization: The objective is to furnish a first estimation $\widehat{\theta}_0$ of the parameters. We used the k-means algorithm.

Parameter estimation: This phase is realized according to the ICE algorithm. It is based on the iterative estimation of the conditional expectation of θ according to $\widehat{\theta}_{n+1} = E_{\widehat{\theta}_n}\left(\widehat{\theta}(X_n, Y)|Y\right)$, where X_n is a realization of X.

Segmentation: The restoration is then achieved using the *Maximum Posterior Mode* (MPM) segmentation rule.

B. Pearson's system of distributions

This Pearson's system consists of a set of height families of distributions, including Gaussian, Gamma and Beta ones. Comprehensive introduction and detailed statements on the Pearson's system are given in [10], [11]. All the families parameters can be expressed in terms of the mean ($\mu = \mu_1$), variance ($\sigma^2 = \mu_2$), skewness ($\sqrt{\beta_1}$ with $\beta_1 = \mu_3^2 \setminus \mu_2^3$) and kurtosis ($\beta_2 = \mu_4 \setminus \mu_2$), which leads to very flexible distribution forms (μ_2, μ_3 and μ_4 denote centered moments).

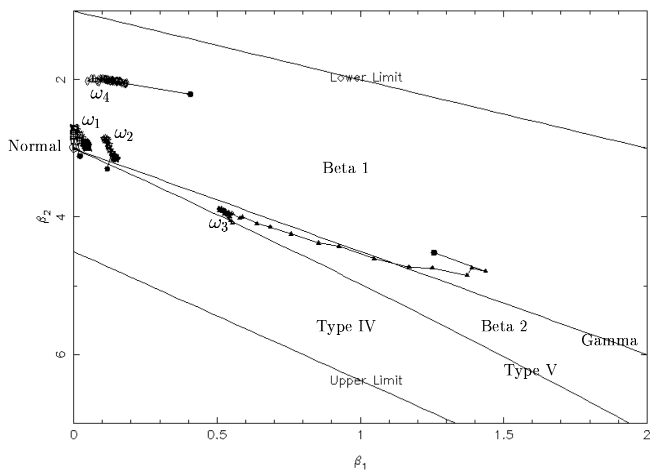


Fig. 2. Pearson's graph (note that the β_2 axis is reversed). The plots represent the iterated estimations of (β_1, β_2) for each class during the ICE process applied on the image in Fig. 1 (see section III).

All the distributions can be in represented in the Pearson's graph (Fig. 2). Gaussian are located in $(\beta_1 = 0, \beta_2 = 3)$, and Gamma are located according to $\beta_2 = 1.5\,\beta_1 + 3$. Beta distributions of the first kind are located below the Gamma line. Beta distributions of the second kind and Type IV (which are very similar to K-distributions) are located above the line.

From a realization x_n of X, one can estimate the empirical moments and compute (β_1, β_2) for each class. Given the graph, it becomes possible to select the corresponding family and recover the parameters which identify the distribution.

III. Experimental results

The algorithm is first illustrated in the framework of classic gaussian mixture estimation and then for generalized mixture estimation (according to Pearson's system). The results we obtained with four classes on the image in Fig. 1 are presented in Fig. 3 and 4.

The upper plots shows the four distributions estimated by 30 ICE iterations. The pathes taken in the (β_1, β_2)-plane during the ICE process are drawn in Fig. 2. The two estimated mixture (see vignettes), i.e. weighted sum of the distributions of the different classes, clearly approaches the overall distribution of the image. Table I shows the estimated distributions and their parameters. All the classes are of beta type in the case of generalized mixture, one can note:

Class 1: is nearly a Gaussian distribution. It represents polynya or thin ice.

Class 2: is nearly a Gamma distribution. It represents brash or pancakes.

Class 3: may represent compact white ice floes.

Class 4: This class is specialized in the histogram queue and acts as a threshold for icebergs.

TABLE I

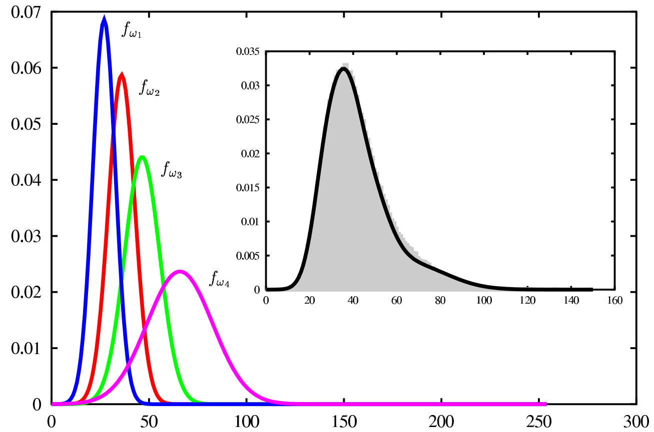
PARAMETER VALUES FOR GENERALIZED AND GAUSSIAN MIXTURES.

ω_k	μ_1	μ_2	μ_3	μ_4	β_1	β_2
1 (Beta 1)	28.8	41.5	59	5132	0.05	2.98
2 (Beta 1)	40.3	67.2	206	14098	0.14	3.12
3 (Beta 2)	60.9	217.2	2317	183472	0.52	3.89
4 (Beta 1)	169.5	1682.9	27234	5771854	0.16	2.04
1 (normal)	27.0	33.9	0	6399	0	3
2 (normal)	35.9	46.2	0	20099	0	3
3 (normal)	46.5	81.9	0	3443	0	3
4 (normal)	65.8	284.3	0	242495	0	3

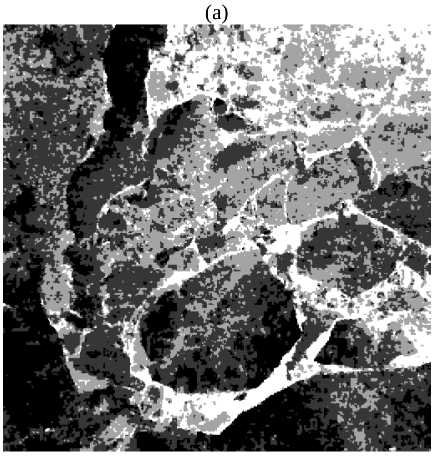
Bayesian decision thresholds are clearly different in the two mixture cases, which results in quiet different segmented images.

IV. Conclusion

The results illustrate the fact that no obvious single model emerged which adequately fit all ice types, and justify the use of generalized mixture estimation. The algorithm described above is not limited to the Pearson's system but can be extended to any

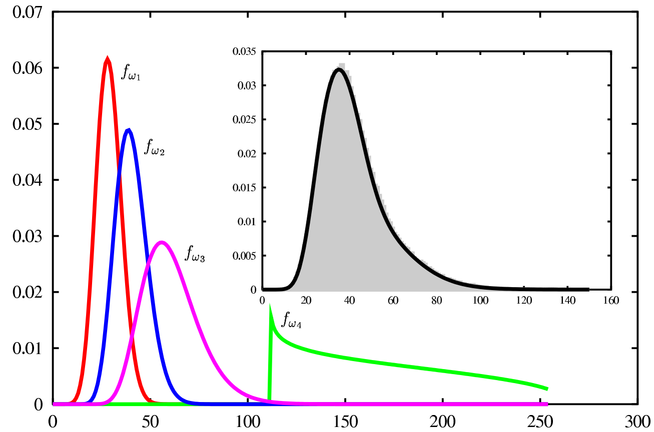


(a)

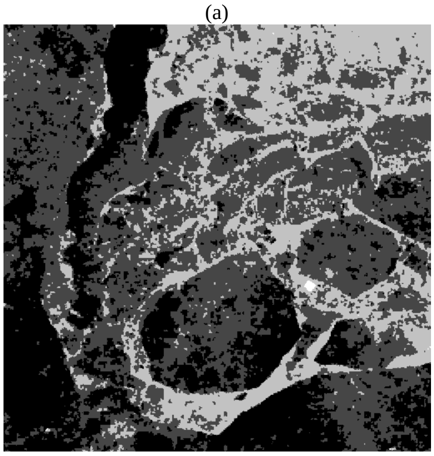


(b)

Fig. 3. Results obtained with a mixture of four Gaussian distributions. (a) plot of the estimated gaussian distributions. The mixture is shown in the vignette. (b) result of segmentation.



(a)



(b)

Fig. 4. Results obtained with a mixture of four distributions from the Pearson's system. (a) plot of the estimated distributions. The mixture is shown in the vignette. (b) result of segmentation. Note the white class f_{w_4} dedicated to the iceberg.

finite sets of distributions [8], using the Kolmogorov distance. This can be useful when backscatter results on the expected densities are known a priori. One interesting point is also to test the specific SAR-oriented KUBW system described in [12] in the context of sea-ice image.

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